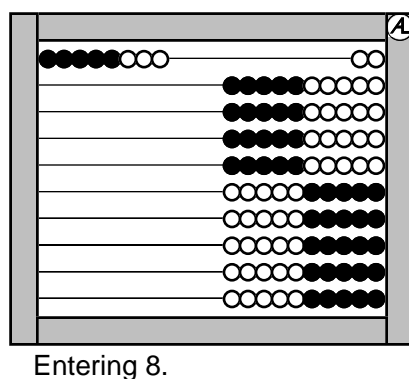
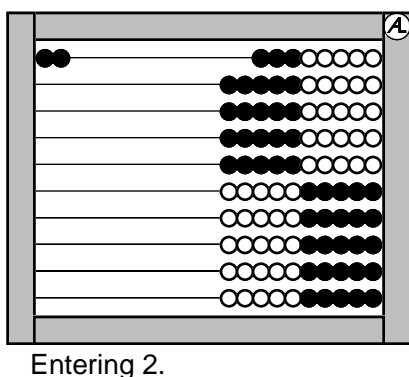
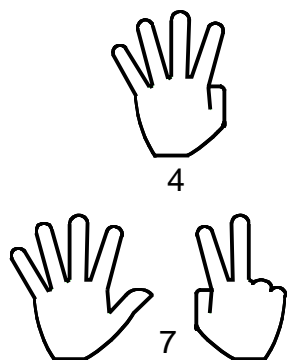


Basics of the AL Abacus

The abacus provides a visual representation and hands-on experience grouping with 5s as well as 10s. I asked Stan, age 5, how much is 11 plus 6. When he said 17 without counting, I asked him how he knew. He answered, "I have the abacus in my mind."

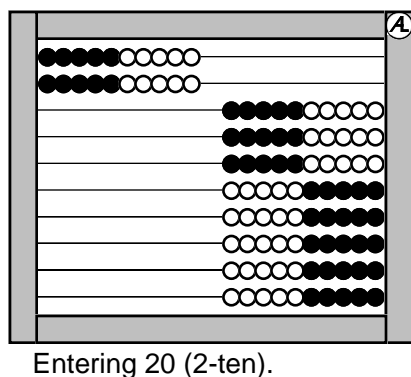
Before teaching children to enter quantities on the abacus, teach them the quantities 1 to 10 with their fingers. Ask them to use the left hand for quantities 1-5 (because we read from left to right) and add the right hand for 6-10. See left figures below.

Set the AL abacus flat on a surface with the dot in the upper right hand corner. Clear by tilting the abacus so the beads fall toward the right. To enter 2, slide two beads together as a unit toward the left. To enter 8, slide all eight beads together to the left. The beads are not to be counted. See the figures below on the right.



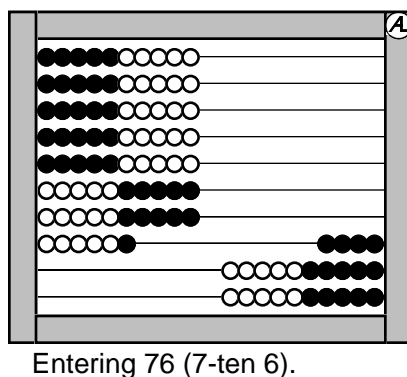
Adding. To add two quantities, such as $4 + 3$, first enter 4 and then add 3. The sum is immediately seen as 7 without any counting.

Tens. Tens are quickly entered because they are entire rows. The colors change after 50 to make it possible to recognize quantities greater than 50. See the examples below. Place-value cards, shown at the right of each figure, show how we write numbers.



2	0
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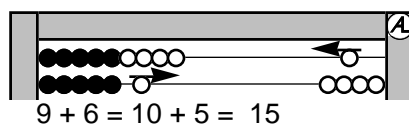
Place-value card for 20.



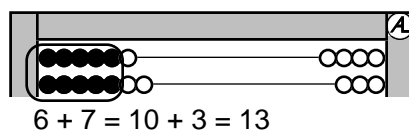
7	0	6
7	6	
7	6	

Composing 76 by overlapping 70 and 6.

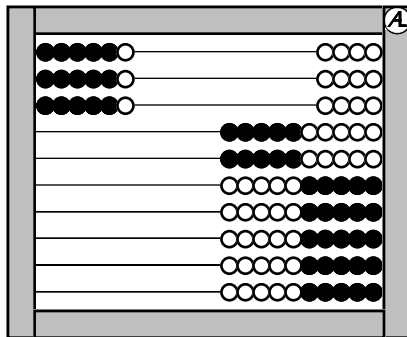
Strategies for the Addition Facts. One addition strategy is Adding Nines. For example, to add $9 + 6$, enter the 9 on the first wire and 6 on the second wire. Then move a bead from the 6 and give it to the 9 to make 10 and 5.



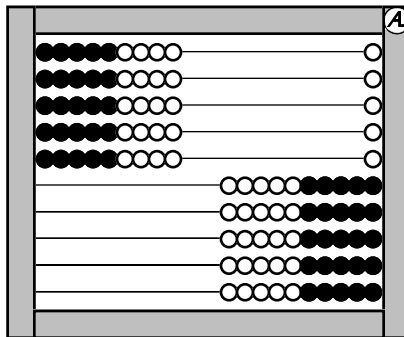
A more interesting strategy is the Two Fives strategy. For example, to add $6 + 7$, enter 6 and 7 on two wires as shown. The two 5s make 10, the remaining 1 and 2 make 3, giving 13. This strategy works for facts when both addends are 5 or more.



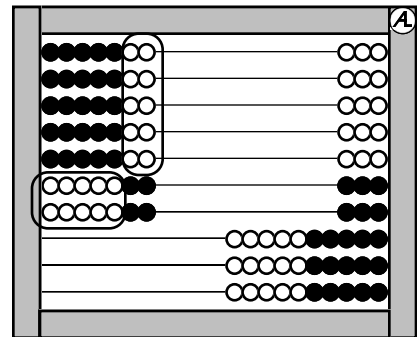
Multiplication. To demonstrate multiplication, enter 6 three times, which is written 6×3 (see the left figure below). The product is seen as $15 + 3$. Next try 9×5 (center figure below); here the product is $10 \times 5 - 5$, or 45. A more interesting example is 7×7 shown below on the right. First see the 5×5 grouping, then the two tens, and finally the last four to give 49.



$$6 \times 3 = 15 + 3 = 18$$



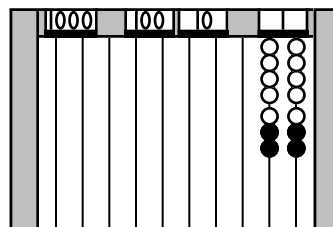
$$9 \times 5 = 50 - 5 = 45$$



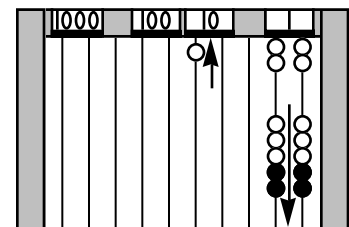
$$7 \times 7 = 25 + 10 + 10 + 4 = 49$$

Side 2 of the AL Abacus. Side 1 of the AL Abacus shows quantities while Side 2 shows procedures. Side 2 has labels above the columns indicating 1000s, 100s, 10s, and 1s. This more abstract and traditional use of the abacus demonstrates *trading* (carrying). Two wires are used for each denomination, which need to be kept as even as possible for quick recognition. The third wire from each end is not used (on this side).

For example, add $8 + 6$; the sum is seen immediately because the two 5s form a 10. But we cannot have 14 ones. To trade use two hands: the right hand moves down ten 1-beads (5 from each wire) while the left hand moves up one 10-bead. See the figures at the right.

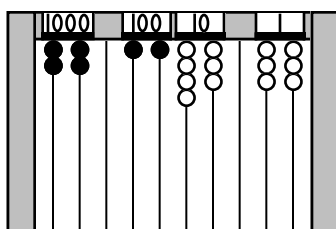


$$8 + 6 = 10 + 4 = 14$$

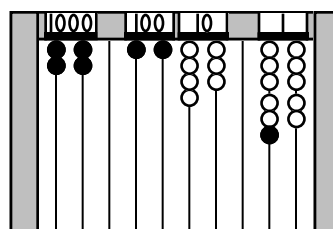


After trading, still 14.

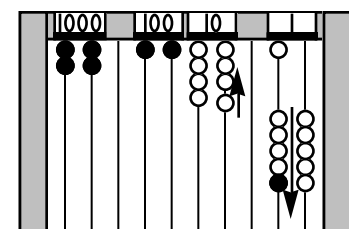
Side 2 Adding. See below for adding $4276 + 445$. It correlates exactly with the usual algorithm.



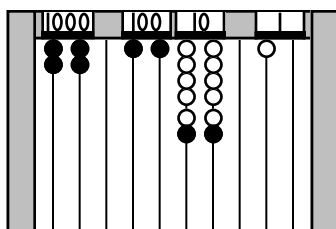
(Entering first quantity.)
$$\begin{array}{r} 4276 \\ + 445 \\ \hline \end{array}$$



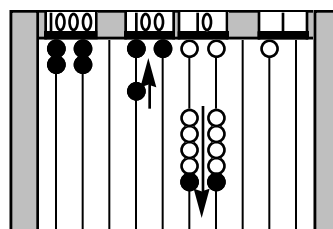
(Adding 5 ones.)
$$\begin{array}{r} 4276 \\ + 445 \\ \hline \end{array}$$



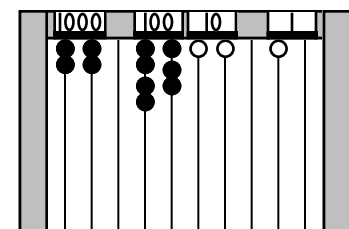
(Trading 10 ones for 1 ten.)
$$\begin{array}{r} 4276 \\ + 445 \\ \hline 1 \end{array}$$



(Adding 4 tens.)
$$\begin{array}{r} 4276 \\ + 445 \\ \hline 1 \end{array}$$



(Trading 10 tens for 1 hundred.)
$$\begin{array}{r} 4276 \\ + 445 \\ \hline 21 \end{array}$$



(Adding 4 hundred.)
$$\begin{array}{r} 4276 \\ + 445 \\ \hline 4721 \end{array}$$