Learning Place Value in First Grade Through Language and Visualization

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International studies, such as the TIMSS studies, show Asian students do better than their American counterparts in mathematics. In the U.S. half the children in fourth grade are still learning place value concepts (Kamii, 1985; Ross, 1989, Miura & Okamoto 1989); whereas, Asian children develop this concept years sooner. There are some valid cultural characteristics favoring Asian students, including a homogeneous population, a longer school year, public value and support of education, and a philosophy of learning that hard work and good instruction, not talent, determine a student's success.

These characteristics are very difficult to change in the U.S.; however, there are some Asian cultural practices that can be implemented: regular value-number naming, visualization rather than counting, and choice of manipulatives, which this study showed could help U.S. children.

Language

One difference is that of naming numbers. Most Asian languages refer to 23, for example, as "2-ten 3" and 67 as "6-ten 7." In English the quantity ten has three names, *ten*, -*teen* and *-ty*. Another confusion are the numbers, 11-19; words eleven and twelve seem to make no sense and for the numbers from 13 to 19, the order is reversed with the ones stated before the tens. All European languages have some irregularities in naming numbers.

Miura and Okamoto (1989) discussed the possibility that the Asian language system of value-naming is one of the factors associated with the high mathematics achievement of Asian-American students. Data from the California Assessment Program (1980, 1982) as cited in Miura and Okamoto showed that Asian-American students scored higher in mathematics than other groups. When data from the 1979-80 year is grouped by language spoken, greater variations were seen. Asian-American third graders who spoke only English scored in the 54th percentile, while students who were also fluent in Chinese or Japanese scored in the 99th and 97th percentiles, respectively (Sells, 1982). This contrasts with bilingual Spanish-speaking third graders who scored in the 16th percentile.

An interesting case is that of the Korean children. A natural experiment in number naming occurs there, because two number systems are spoken. For everyday, or informal, speech the number words have irregularities, but the formal number system used in school is value-named and completely regular. No words are the same in the two systems. Korean children trailed U.S. children in their ability to count at age 4, (Song and Ginsburg, 1988). See Fig. 1. However, at age 5 when they learned the regular system, their counting ability rose rapidly in both systems. The curve of the U.S. children continued at the same rate, indicating rote memorization.

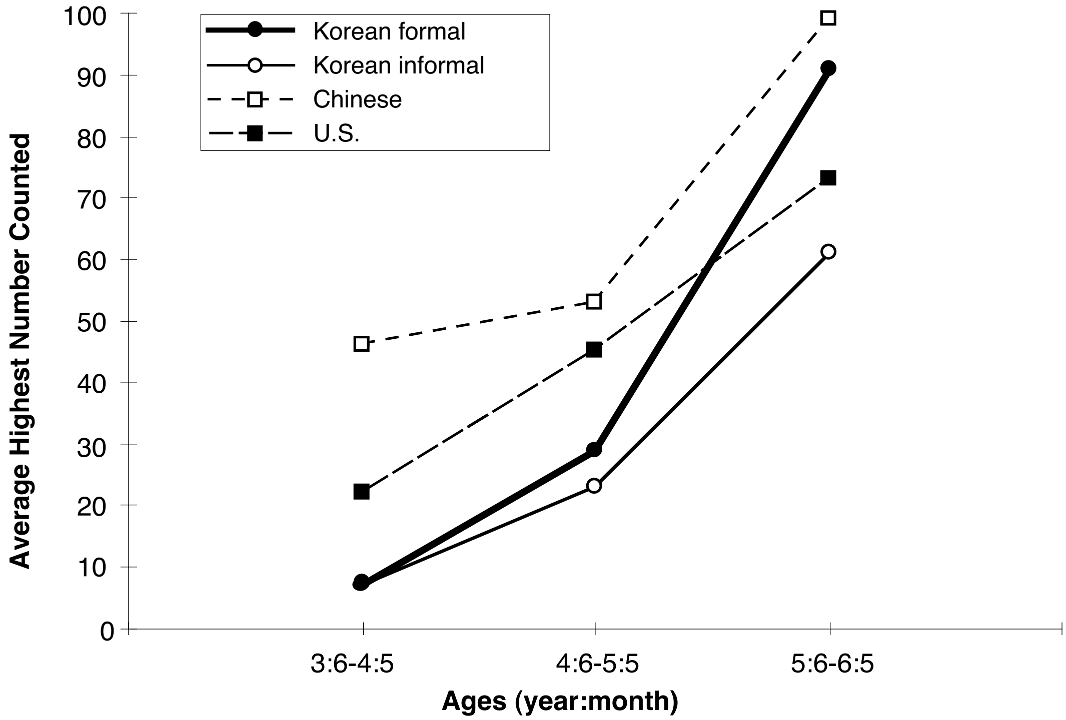


Figure 1. Counting ability by language.

Chart from Song and Ginsburg (1988) p. 326.

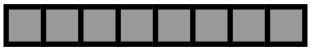
Song, M., & Ginsburg, H. (1988). The Effect of the Korean Number System on Young Children's Counting: A Natural Experiment in Numerical Bilingualism. *International Journal of Psychology,* 23, pp. 319-332.

Visualization vs. Counting

Another major difference is the view of counting. In the U.S. counting is considered the basis of arithmetic; children engage in various counting strategies: counting all, counting on, and counting back. Conversely, Japanese children are discouraged from counting; they are taught to recognize and visualize quantities in groups of fives and tens. Children using counting, which is slow and often unreliable, to add and subtract develop a unitary concept of number. For example, they think of 14 as 14 ones, not as 1 ten and 4 ones. Such thinking interferes with understanding carrying and borrowing in larger numbers.

To understand the importance of visualization, try to see mentally 8 apples in a line without any grouping–virtually impossible. Now try to see 5 of those apples as red and 3 as green; the vast majority of people can form the mental image. The Japanese employ this sub-base of 5 to make quantities between 6 and 10 easily imaginable. Thus, 8 is seen as 5 and 3. See Fig. 2.

Also, Japanese primary classrooms have very few manipulatives, all of which the children must be able to visualize; in contrast to U.S. classrooms, which usually have an abundance of manipulatives.



This collection needs to be counted. It cannot be visualized.

This collection can be recognized and visualized, without counting.

Figure 2.

The Study

Research was conducted in an experimental first grade classroom of 16 children in a rural community in Minnesota, USA, during the 1994-95 school year (Cotter, 1996). A matched class, the control, was taught in the traditional workbook method. The researcher supplied lesson plans in the experimental class.

All mathematical activities concerning quantities centered on place value. Naming quantities, representing them concretely and pictorially, computing, and recording, all focused on ones, tens, hundreds, and thousands.

The study included six major components: (a) visualizing quantities, (b) value-naming of tens and ones, (c) an abacus displaying a sub-base of five, (d) overlapping place value cards, (e) part-part-whole partitioning, and (f) early introduction of multidigit addition and subtraction. Only the latter two components had been studied previously.

The AL Abacus

A specially designed double-sided abacus, called the AL Abacus, allowed the children to represent quantities based on fives and tens. On Side 1 of the AL abacus, each bead has a value of 1. See Figure 3. There are 10 wires, each with 2 groups of 5 beads in contrasting colors. The first 5 rows have 5 dark colored-beads followed by 5 light-colored beads. The two colors allow instant recognition, so counting is not needed. Quantities are considered "entered" when they are moved to the left side.

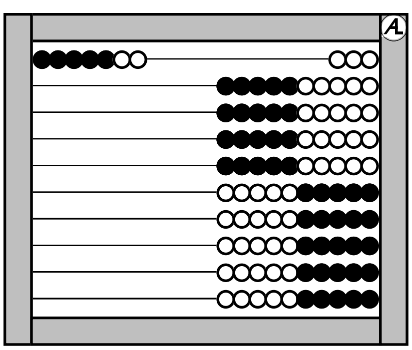


Figure 3. Representing 7 on side 1 of the AL abacus.

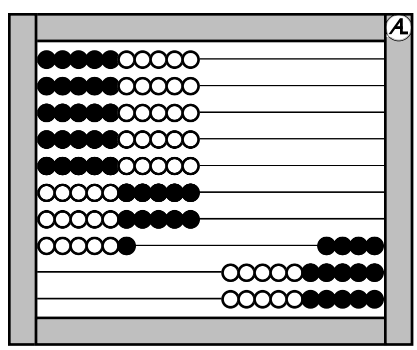


Figure 4. The quantity 76 entered on the abacus.

The last 5 rows are reversed: 5 light-colored beads followed by 5 dark-colored beads, permitting instant recognition of more than 5 tens. See Fig. 4. Thus, any quantity from 1 to 100 can be visualized and recognized. Hundreds are built by combining several abacuses. For example, stacking 3 abacuses represents 300 and stacking 10 abacuses represents 1000.

Visualization also played a part in strategies for learning the facts. For example, to add 9 + 4, 1 is removed from the 4 and combined with the 9 to give 10 and 3, or 13.

On Side 2 of the abacus, beads have a value according to their position. See Figure 5. Note that two wires are used to show each denomination. Enter quantities by moving beads up. The children worked with four-digit quantities, trading between denominations as needed.

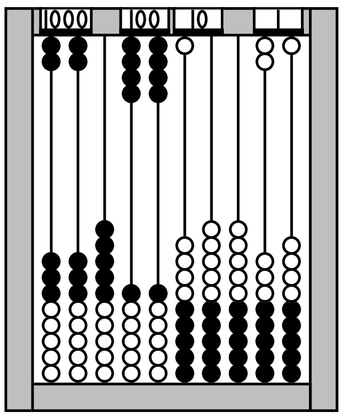


Figure 5. Representing 4813 on side 2 of the AL abacus.

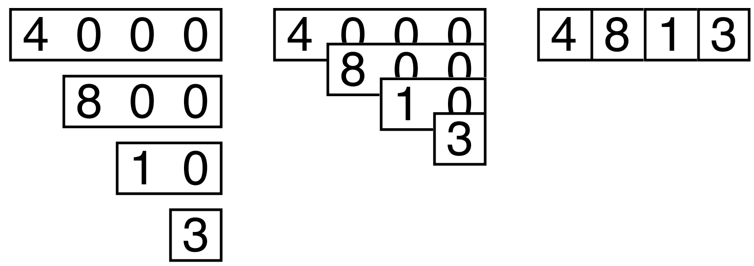


Figure 6. Overlapping place value cards showing the composition of 4813.

Place Value Cards

To help children compose and record multidigit numbers, they used overlapping place value cards. See Figure 6. Children learned the 8 in 813 is 8 hundred because two zeroes (or other digits) follow it. This meant they read numbers in the normal left to right order, and not backwards as is done with the column approach of starting at the right and saying, "ones, tens, hundreds."

Results

Some significant findings comparing the experimental class to the control class are the following: (a) Three times as often, the experimental class preferred to represent numbers 11, 13, 28, 30, and 42 with tens and ones instead of a collection of ones. (b) Only 13% of the control class, but 63% of the experimental class correctly explained the meaning of the 2 in 26 after the 26 cubes were grouped in 6 containers with 2 left over. (c) In the control class 47% knew the value of 10 + 3 and 33% knew 6 + 10, while 94% and 88%, respectively, of the experimental class knew. (d) In the control class 33% subtracted 14 from 48 by removing 1 ten and 4 ones rather than 14 ones; 81% of the experimental class did so. (e) When asked to circle the tens place in the number 3924, 7% of the control class and 44% of the experimental did so correctly. (f) None of the control class mentally computed 85 – 70, but 31% of the experimental class did. (g) For the sum of 38 + 24, 40% of the control class incorrectly wrote 512, while none in the experimental class did.

Notable comparisons with the work of other researchers showed that: (a) all of the children in this study made at least one “tens and ones” representation of 11, 13, 28, 30, and 42, while only 50% of the U.S. children did so in the study by Miura & Okamoto (1989); (b) 63% of children in this study made all five “tens and ones” representations, while only 2% of the U.S. children did so in the study by Miura & Okamoto; (c) 93% of the children explained the meaning of the digits in 26 while 50% of the third graders in Ross’s (1989) did so; (d) 94% of the children knew 10 + 3 while 67% of beginning second graders in Steinberg’s (1985) knew; (e) 88% knew 6 + 10 compared to 72% of the second graders in Reys et al. (1993) study; (f) 44% of the children circled the tens place in 3924 while data from the 1986 NAEP (Kouba et al., 1988) found 65% of third graders circled the tens place in a four-digit number; (g) 63% of the children named 511 as greater than 298, which compared to 40% of 6-year-olds in Geneva, Switzerland, and 33% in Bariloche, Argentina, (Sinclair & Scheuer, 1993); (h) 56% mentally computed 64 + 20, which compared to 52% of nine-year-olds on the 1986 NAEP study; and (i) 69% mentally computed 80 – 30 while 9% of the second graders in Reys et al. study did so.

The children also worked with four-digit addition and subtraction algorithms. They learned the procedure on side 2 of the abacus and spontaneously transferred their knowledge to the paper and pencil algorithm. While learning the procedure, they recorded their results as it was formed on the abacus along with any carries. The children did not practice the algorithm for two-place numbers as these were done mentally. On the final test where the problem, 2304 + 86 =, was written horizontally, 56% of the children did it correctly, including one child who did it in his head.

Summary

Both the teacher and children enjoyed this new approach for first grade mathematics. The children did construct a tens-base approach to numbers, rather than a unitary concept. They learned their addition and subtraction combinations through strategies based on fives and tens.

The lowest ability child, who weighed about 1200 gm (2 pounds 9 ounces) at birth and was hydrocephalic, was asked to draw what 12 looks like. He drew 10 objects in the first row and 2 in the second row and explained it by saying that it had to be that way because 12 is 10 and 2. He also could mentally add 9 to a number; for example, he added 9 + 4 by changing it to 10 + 3. The most advanced child at the conclusion of the study was surprised to learn that not all children learned to add and subtract 4-digit numbers in first grade. There were no problems or complaints from parents with the children using value-named words for numbers, which they did for the first three months of the school year.

Sequel

The following year, 1995-96, the lesson plans were modified and used for both first grade classes. In April, both classes took the First Grade Testronics National Standardized Test, published by ACT (American College Test), and scored at the 98th percentile. The program was also introduced into the kindergarten. Half of the children developed the concept of tens and ones.

In the 1996-97 school year, there was one first grade class with 23 children; that class also scored at the 98th percentile.

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Electronic Abacus: http://illuminations.nctm.org/mathlets/abacus/index.html

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